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Functions Feasibility Analysis: Based on Cardinality of Sets

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Abstract— this paper gives a precise yet easy generalization corresponding to functions based on the cardinality [3] of sets on which set is defined. Let f : A→B, where A,B are finite and non-empty sets such that │A│=m,│B│= n, m and n are integer values. This paper justify the existence of different kind of algebraic functions like one-one, onto, into, many-one, Bijective etc depending on the relation between m and n.

Keywords— **function [3], cardinality [3], into, onto (surjective), bijective, one-one, mapping [1], transformation [2].**

I. Introduction

The concept of a function is extremely important in mathematics and computer science. For example, in discrete mathematics functions are used in the definition of such discrete structures as sequences and strings. Functions are also used to represent how long it takes a computer to solve problems of a given size. Many computer programs and subroutines are designed to calculate values of functions.

In many instances we assign to each element of a set a particular element of a second set For example, suppose that each student in a discrete mathematics class is assigned a letter grade from the set {A, B, C, D}, and suppose the grades are A for ST1, B for ST2, C for ST3, A for ST4, C for ST5. This assignment of grades is shown below:

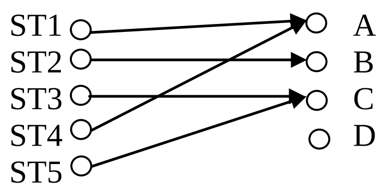

Fig 1: Assignment of grades to students

Here
f(ST1)=A, f(ST2)=B, f(ST3)=C, f(ST4)=A, f(ST5)=C

Definition: Let *A* and *B* be nonempty sets. A *function f* from *A* to *B* is an assignment of exactly one element of *B* to each element of *A*. We write *f (a) = b* if *b* is the unique element of *B* assigned by the function *f* to the element *a* of *A*. If *f* is a function from *A* to *B*, we write

f :A→B. In general f (a) =b, ∇ a∈A, b∈ *B*
Functions are sometimes called mappings or transformations also.

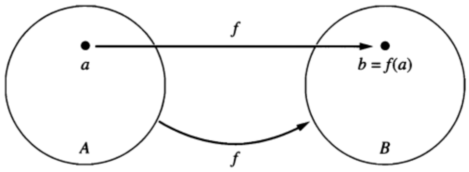

Figure 2: Function *f* maps A to B

II. Feasibility Analysis for Various Types of Functions

Since functions are simply the assignment of elements of set B to the elements of set A. Now this assignment of elements can be done in different manners depending on the constructs in use like: Some functions never assign the same value to two different domain elements. These functions are said to be one-to-one, in some of the cases we can assign the same element from set B to number of elements of set A. So based on the assignment strategy in use many feasible functions are:

1. One-one(injective)
2. Into
3. Onto(surjective)
4. Many-one
5. One-one onto(Bijective) etc

Let f: A→B be a function from set A to B.

A. *One-One function*

These functions never assign the same value to two different domain elements. Note that a function *f* is one-to-one if and only if $f(a) \neq f(b)$ whenever $a \neq b$.

Feasibility check w.r.t. cardinality such that $|A|=m$, $|B|=n$

1. If m<n, an one-one function can be designed from A to B

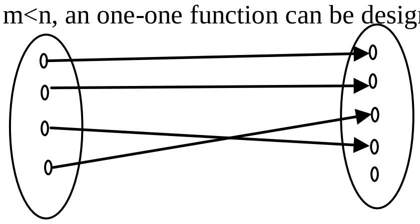

Fig 3: one-one function

2. If m=n,

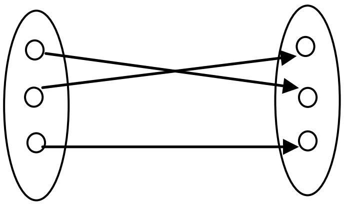

Fig 4: one-one function

3. If m>n, no one-one function is feasible from A to B.

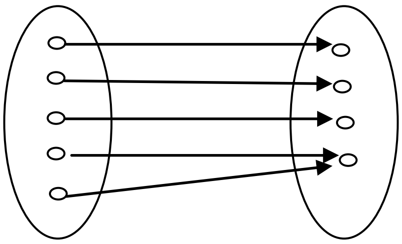

Fig 5: one-one function

Here more than one elements of A have been assigned the same element of set B, so this is not a one-one function. So if m>n, there will never be a onto function from A to B.

Result:
An one-one function from A to B is feasible iff

1. m<n
2. m=n

$$f: A \rightarrow B \begin{Bmatrix} an\ one-one\ iff\ m \leq n \\ else\ not\ an\ onto\ function \end{Bmatrix}$$

B. Onto function

Here every element of set B is assigned to at least one of the element of set A.
A function *f* from *A* to *B* is called *onto*, or *surjective* [2], if and only if for every element $b \in B$ there is an element $a \in A$ with $f(a) = b$. A function *f* is called a *surjection* if it is onto.

Example:

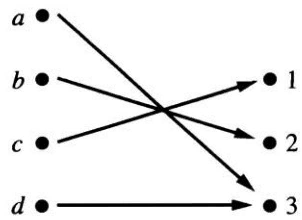

fig 6: onto function

Feasibility check:

1. If m<n, an onto function can not be designed from A to B, because one element of A can be assigned only one element of B as shown below:

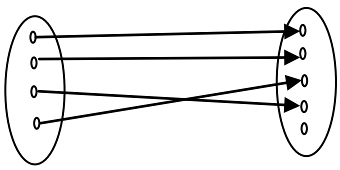

Fig 7: onto function

So, if m<n onto function is not feasible from A to B.

2. If m=n

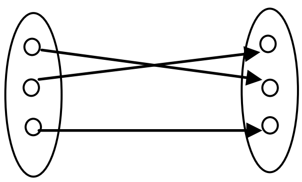

Fig 8: onto function

3. If m>n, onto function is feasible from A to B.

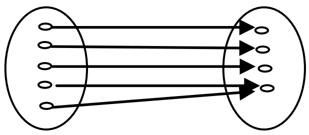

Fig 9: onto function

Result:
An onto function from A to B is feasible iff

1. m>n
2. m=n

hence:

$$f: A \rightarrow B = \begin{Bmatrix} will\ be\ an\ onto\ function\ iff\ m \geq n \\ else\ no\ onto\ function\ can\ be\ desined \end{Bmatrix}$$

C. In-to function:

Here every element of set B is not assigned to the elements of set A. Some of the elements of B may not be the image of any of the elements on set A.

A function f from A to B is called *into*, if and only if $\exists\, b \in B$ such that $f(a) \neq b$.

Example:

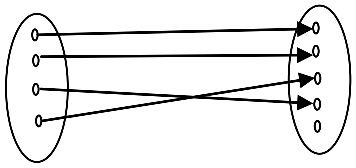

Fig 10: into function

Here one of the elements of set B is not assigned to any of the element of set A.

Feasibility Check:

Case 1: if m<n

An Into function can be designed that may or may not be one-one as well as shown below:

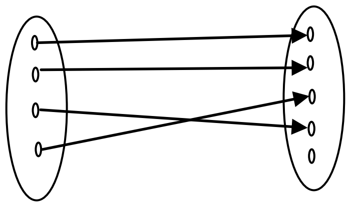

Fig 11: into function

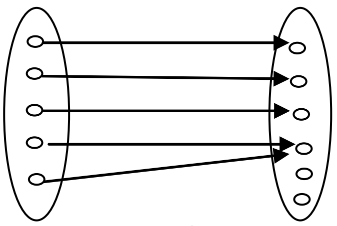

Fig 12: into function

Case 2: m=n

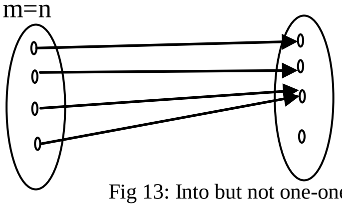

Fig 13: Into but not one-one

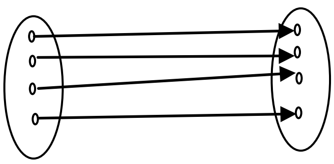

Fig 14: Not into but one-one

Case 3: m>n

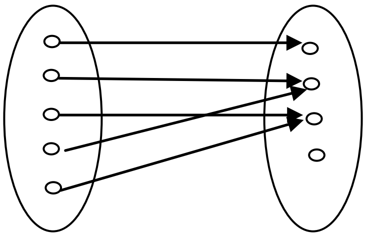

Fig 15: Into but not one-one

Similarly we can show that if m>n no function can be designed that is both into as well as one-one.

$$f: A \rightarrow B \text{ will be } \begin{cases} \text{one to one as well into if } m < n \\ \text{one} - \text{one as well as many} - \text{one if } m < n \\ \text{into but not one} - \text{one if } m = n \\ \text{one} - \text{one but not into if } m = n \\ \text{into but not one} - \text{one if } m > n \end{cases}$$

D. Bijective function:

A function is said to be bijective iff it is One-One as well as onto function.
Example:

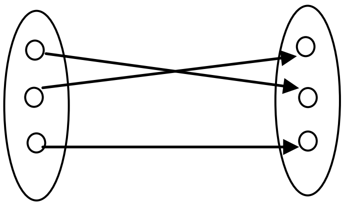

Fig 16: Bijective function

Feasibility check:

Case 1: m<n
One-one function can be designed but not an onto function so there is no bijective function feasible in this case.

Case 2: m=n

Bijective function is feasible as shown:

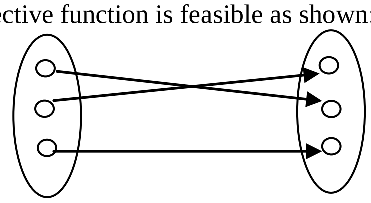

Fig 17: Bijective function

Case 3: m>n
No one-one function can be designed, so no bijective function also.

Hence

$$f: A \to B \begin{Bmatrix} \text{will be a bijective function iff } m = n \\ \text{else if } m < n \text{ or } m > n \text{ no bijetive function can be designed} \end{Bmatrix}$$

E. Inverse function

Let $f: A \to B$ be a function, its inverse function $g: B \to A$ will be feasible iff f is onto as well as one-one.

So for a function $f: A \to B$ inverse function exits iff f is an one-one and onto function i.e. a bijective function. So, based on the results concluded above

$$\bar{f}: B \to A \begin{Bmatrix} \text{if } m = n, \text{feasible} \\ \text{else no inverse function can be designed} \end{Bmatrix}$$

III. TABLES

TABLE I
Functions f: A→B feasibility check where
|A| = m and |B| = n

Function / case	Onto	Into	One-one	Bijective	Inverse
m<n	No	Yes	Yes	No	No
m=n	Yes	Yes	Yes	Yes	Yes
m>n	Yes	Yes	No	No	No

IV *Conclusions*

Based on above results we can easily check the feasibility of various types of functions from a set A to B provided A and B are finite and non-empty sets with cardinality m and n respectively. By simply checking the relationship between m and n we can state that which function is feasible and which is not. So it is precise and easy to understand generalization of functions feasibility between two sets.

V References

[1] K. Rosen, Discrete Mathematics and its Applications (6th ed), New York: McGraw-Hill, 2007.
[2] L. Gerstein, Discrete Mathematics and Algebraic Structures, New York: Freeman and Co., 1987.
[3]C L Liu, D P Mohapatra, Elements of Discrete Mathematics, TMH, 2008
[4] Richard Johnsonbaugh, *Discrete Mathematics*, Prentice Hall, 2008.