



## EFFECTIVE CONGESTION CONTROL IN WIRELESS AD-HOC NETWORKS

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**Abstract**—we study joint end-to-end congestion control and Per-link medium access control (MAC) in ad-hoc networks. We use a network utility maximization formulation, where the goal is to find optimal end-to-end source rates at the transport layer and per-link persistence probabilities at the medium access control (MAC) layer to maximize the aggregate source utility. Under certain conditions, by applying appropriate transformations and introducing new variables, we obtain a decoupled and dual-decomposable convex formulation. We develop a novel dual-based distributed algorithm using the sub gradient method. In this algorithm, sources at the transport layer adjust their log rates to maximize their net benefits, while links at the MAC layer select transmission probabilities proportional to their conceived contribution to the system reward. The two layers are connected and coordinated by link prices.

**Keywords**—Ad-hoc networks, random access, wireless networks, MAC (medium access control), congestion

### 1. INTRODUCTION

AD-HOC wireless networks are usually defined as an autonomous system of nodes connected by wireless links and communicating in a multi-hop fashion. The benefits of ad-hoc networks are many, but the most important one is their ease of deployment without centralized administration or fixed infrastructure, thereby enabling an inexpensive way to achieve the goal of ubiquitous communications. One of the fundamental tasks that an ad hoc network should often perform is congestion

control. Congestion control is the mechanism by which the network bandwidth is distributed across multiple end-to-end connections. Its main objective is to limit the delay and buffer overflow caused by network congestion and provide tradeoffs between efficient and fair resource allocation.

Congestion is an unwanted situation in networked systems. Congestion can be disastrous for a data transmission system as it manifests itself as depletion of resources that are critical to the operation of the system. These resources can be CPU, buffer space, bandwidth etc. Resource crunch will lead to lengthening of various queues for these resources. “Congestion control” refers to the mechanism of combating congestion, which makes sure the resources are used optimally and the system has maximum data throughput with the given conditions. The main objective of congestion control is to make sure the system is running at its rated capacity, even with the worst case overload situations. In certain systems, this is ensured by restricting certain nodes to transmit at the maximum capacity or to make use of certain resources monotonously. Doing this enables optimal usage of resources for all the nodes in the system with a measurable quality-of-service (QOS). In some systems, there are built-in mechanisms that does not allow congestion situation to take place and every node keeps track of system statistics and resources. This is often known as “congestion prevention” or “Congestion avoidance”.

Unlike in wire-line networks where links are disjoint resources with fixed capacities, in ad hoc wireless networks the link capacities are “elastic”.

Most routing schemes for ad hoc networks select paths that minimize hop count. This implicitly predefines a route for any source-destination pair of a static network, independent of the pattern of traffic demand and interference among links. This may result in congestion at some region, while other regions are not fully utilized. To use the wireless spectrum more efficiently, multiple paths based on the pattern of traffic demand and interference among links should be considered.

Wireless channel is a shared medium and interference limited. Link is only a logical concept and links are correlated due to the interference with each other. Under the MAC strategies such as time-division and random access, these links contend for exclusive access to the physical channel. Unlike in the wire-line network where network layer flows compete for transmission resources only when they share the same link, in wireless network flows can compete even if they don't share a wireless link in their paths. Thus, in ad hoc wireless networks the contention relations between link-layer flows provide fundamental constraints for resource allocation.

In this TCP congestion control algorithms can be considered as distributed primal-dual algorithms which maximize aggregate network utility, where a user's utility function is defined by its TCP algorithm. These works implicitly assumes a wire-line network where link capacities are fixed and shared by flows that traverse common links. In wireless networks the joint design of congestion and media access control is naturally formulated using the network utility maximization framework considering the new constraints that arise from channel contention. In wire line networks, congestion control is implemented at the transport layer and is often designed separately from functions of other layers. Since wired links have fixed capacities and are independent, this methodology is well justified and has been studied extensively. In recent years, useful mathematical models and tools based on convex optimization and control theory have been developed, which cast congestion control algorithms as decentralized primal-dual schemes to solve network utility maximization problems.

In the NUM framework, each end-user (or source) has its utility function and link bandwidths are allocated so that network utility (*i.e.*, the sum of all users' utilities) is maximized. A utility function can be interpreted as the level of satisfaction attained by a user as a function of resource allocation. Efficiency of resource allocation algorithms can thus be measured by the achieved network utility.

## 2. MODELING AND PRELIMINARIES

We consider an ad hoc wireless network represented by an undirected graph  $G = (N, L)$ , where  $N$  is the set of nodes and  $L$  is the set of logical links. Each source node  $s$  has its utility function  $U_s(x_s)$ , which is a function of its transmitting data rate  $x_s \in [0, \infty)$  and we assume it is continuously differentiable, increasing, and strictly concave. For its communication, each source uses a subset  $L(s)$  of links. Let  $L_{out}(n)$  denote the set of outgoing links from node  $n$ , and  $L_{in}(n)$  the set of incoming links to node  $n$ . We define  $S$  as the set of all sources and  $S(l)$  as the subset of sources that are traversing link  $l$ . We assume static topology (the nodes are in a fixed position). Also, each link has finite capacity  $cl$  when it is active, *i.e.* we implicitly assume that the wireless channel is fixed or some underlying mechanism masks the channel variation. Wireless transmissions are interference-limited. All links transmit at rate  $cl$  for the duration they hold the channel. Assume that each node cannot transmit or receive simultaneously, and can transmit to or receive from at most one adjacent node at a time. Since each node has a limited transmission range, contention among links for the shared medium is location-dependent. Spatial reuse is possible only when links are sufficiently far apart. Define two types of sets,  $LI(n)$  and  $NI(l)$ , to capture the location dependent contention relations, where  $LI(n)$  is the set of links whose receptions are affected adversely by the transmission of node  $n$ , excluding outgoing links from node  $n$ , and  $NI(l)$  is the set of nodes whose transmission fail the reception of link  $l$ , excluding the transmit node of link  $l$ . Also note that  $l \in LI(n)$  if and only if  $n \in NI(l)$ . Time is slotted in intervals of equal unit length and the  $i$ -th slot refers to the time interval  $[i, i + 1)$ , where  $i = 0, 1, \dots$  *i.e.*, transmission attempts of each node occur at discrete time instances  $i$ . In this a MAC protocol is developed based on random access with probabilistic (re-)transmissions. At the beginning of a slot, each node  $n$  transmits data with probability  $q_n$ . When it determines to transmit data, it selects one of its outgoing links  $l \in L_{out}(n)$  with probability  $p_l/q_n$ , where  $p_l$  is the link persistence probability;

$$\sum_{l \in L_{out}(n)} p_l = q_n, \forall n$$

$$\begin{aligned}
& \max \sum_s U_s(x_s) \\
& \text{s.t. } \sum_{s \in S(l)} x_s \leq \eta_l := c_l p_l \prod_{k \in N^I(l)} (1 - q_k), \forall l \\
& \quad \sum_{l \in L_{out}(n)} p_l = q_n \leq 1, \forall n \\
& \quad 0 \leq p_l \leq 1, \forall l, \\
& \quad x_s \geq 0, \forall s,
\end{aligned} \tag{1}$$

Where  $\eta = \{\eta_l | l \in L\}$  link throughputs given  $p$  and  $q$ , since the term  $p_l \prod_{k \in N^I(l)} (1 - q_k)$  is the probability that a packet is transmitted over link  $l$  and successfully received by its receiver.

### Random models

At physical level we simulate the wireless ad hoc network by placing the nodes geographically random. While at communication level for the communications among the end users, we consider several types of network models: random, scale-free, small-world, star, geographically random and full mesh. The random model can be considered as the most basic model of complex networks. A random network is obtained by starting with a set of  $n$  vertices and randomly adding edges between them. Most commonly studied is the Erdős–Rényi model, denoted  $G(n, p)$ , in which every possible edge occurs independently with probability  $p$ . The degree distribution  $pk$  (the fraction of nodes having  $k$  links) is a Poisson distribution.

The problem formulation in (1) entails congestion control at the network layer (finding  $x$ ), and contention control at the MAC layer (finding  $p$  and  $q$ ). The two layers are coupled through the first constraint in (1), which asserts that for each link  $l$ , the aggregate source rate  $\sum_{s \in S(l)} x_s$  does not exceed the link throughput. The transport layer source rates and the MAC layer transmission probabilities should be jointly optimized to maximize the aggregate source utility. Due to the first constraint, (1) is in general a non-convex and non-separable problem, which is difficult to optimize over both  $x$  and  $p, q$  in a distributed way directly. Under certain conditions, it can be transformed into a convex one by taking the logarithm on both sides of the first constraint and replacing the rate variables by their logarithmic counterparts, i.e.,  $Z_s = \log(x_s)$ . This yields a new constraint

$$\begin{aligned}
& \log\left(\sum_{s \in S(l)} e^{z_s}\right) - \log(c_l) \\
& \quad - \log p_l - \sum_{k \in N^I(l)} \log(1 - q_k) \leq 0, \forall l. \tag{2} \\
& \log\left(\sum_{s \in S(l)} e^{z_s}\right), \text{ although it is a convex function.}
\end{aligned}$$

We introduce a set of new variable

where each  $\alpha_l s$  can be interpreted as the fraction of the overall traffic on link  $l$  contributed by source  $s$ .

$$x_s \leq \alpha_{ls} c_l p_l \prod_{k \in N^I(l)} (1 - q_k), \forall s \in S(l). \tag{3}$$

$$\begin{aligned}
& \text{s.t. } z_s - \log \alpha_{ls} - \log c_l - \log p_l \\
& \quad - \sum_{k \in N^I(l)} \log(1 - q_k) \leq 0, \forall l, \forall s \in S(l) \\
& \quad 0 \leq \alpha_{ls} \leq 1, \sum_{s \in S(l)} \alpha_{ls} = 1, \forall l, \forall s \in S(l) \\
& \quad \sum_{l \in L_{out}(n)} p_l = q_n \leq 1, \forall n \\
& \quad 0 \leq p_l \leq 1, \forall l,
\end{aligned} \tag{4}$$

*Lemma 1:* If  $g_s(x_s) < 0$ , then  $U_s$  ( $Z_s$ ) is a strictly concave function of  $Z_s$ .

$$g_s(x_s) = \frac{d^2 U_s(x_s)}{dx_s^2} x_s + \frac{U_s(x_s)}{dx_s}, \tag{5}$$

Given that the condition of Lemma 1 is satisfied, problem (4) is indeed a convex problem, and all log rates are decoupled, enabling the dual decomposition. To proceed, we apply duality theory and associate Lagrange multipliers. Let us define the Lagrangian function

$$\begin{aligned}
L(\lambda, \mathbf{z}, \mathbf{p}, \mathbf{q}, \alpha) &= \sum_{s \in S} U'_s(z_s) \\
& - \sum_{l \in L} \sum_{s \in S(l)} \lambda_{ls} \left( z_s - \log[\alpha_{ls} c_l p_l \prod_{k \in N^I(l)} (1 - q_k)] \right) \\
& = \sum_{s \in S} \left( U'_s(z_s) - \lambda^s z_s \right) + \sum_{l \in L} \lambda^l \log c_l \\
& + \sum_n \sum_{l \in L_{out}(n)} \sum_{s \in S(l)} \lambda_{ls} \log \alpha_{ls} \\
& + \sum_n \left( \sum_{l \in L_{out}(n)} \lambda^l \log p_l + \sum_{l \in L^I(n)} \lambda^l \log(1 - q_n) \right),
\end{aligned} \tag{6}$$

where  $\lambda^s := \sum_{l \in L(s)} \lambda_{ls}$ ,  $\lambda^l := \sum_{s \in S(l)} \lambda_{ls}$ ,  $\lambda := \{\lambda_{ls} | s \in S, l \in L(s)\}$ , and  $\mathbf{z} := \{z_s | s \in S\}$ .

The Lagrangian dual function is

$$D(\lambda) = \max_{\substack{0 \leq \alpha_{ls} \leq 1, \sum_{s \in S(l)} \alpha_{ls} = 1, \forall l, \forall s \in S(l) \\ \sum_{l \in L_{out}(n)} p_l = q_n \leq 1, \forall n \\ 0 \leq p_l \leq 1, \forall l}} L(\lambda, \mathbf{z}, \mathbf{p}, \mathbf{q}, \alpha), \tag{7}$$

and the dual problem to (4) is

$$\mathbf{D} : \min_{\lambda \geq 0} D(\lambda). \tag{8}$$

The maximization in (7) for a given  $\lambda$  can be decomposed into three sub problems:

One at each source and the other two at each node. The source sub problem is

$$\max_{z_s} (U'_s(z_s) - \lambda^s z_s), \forall s \in S. \quad (9)$$

If we interpret the Lagrange multiplier  $\lambda l_s$  as the price per unit of log bandwidth charged by link  $l$  to source  $s$ , then the source strategy is to maximize its net benefit  $U_s(Z_s) - \lambda_s Z_s$ , since  $\lambda_s z_s$  is just the sum bandwidth cost charged by all links on its path if source  $s$  transmits at log rate  $z_s$ . Since  $U_s(Z_s)$  is strictly concave over  $Z_s$ , a unique maximize exists.

The other two subproblems at each node  $n$  for every outgoing link  $l \in L_{out}(n)$  are, respectively

$$\max_{0 \leq \alpha_{l_s} \leq 1, \sum_{s \in S(l)} \alpha_{l_s} = 1} \sum_{s \in S(l)} \lambda_{l_s} \log \alpha_{l_s}, \forall l \in L_{out}(n), \quad (10)$$

and

$$\max_{\substack{\sum_{l \in L_{out}(n)} p_l = q_n \leq 1 \\ 0 \leq p_l \leq 1}} \sum_{l \in L_{out}(n)} \lambda^l \log p_l + \sum_{l \in L^I(n)} \lambda^l \log(1 - q_n). \quad (11)$$

*Proposition 2:* Given  $\lambda$ , the  $\alpha(\lambda)$  solving problem (10) is (for node  $n$  and link  $l \in L_{out}(n)$ )

$$\alpha_{l_s}(\lambda) = \begin{cases} \frac{\lambda_{l_s}}{\sum_{s \in S(l)} \lambda_{l_s}}, & \text{if } \sum_{s \in S(l)} \lambda_{l_s} \neq 0 \\ \frac{1}{|S(l)|}, & \text{if } \sum_{s \in S(l)} \lambda_{l_s} = 0 \end{cases}, \quad (12)$$

and the  $p(\lambda), q(\lambda)$  solving problem (11) are

$$p_l(\lambda) = \begin{cases} \frac{\lambda^l}{\sum_{l' \in L_{out}(n)} \lambda^{l'} + \sum_{l' \in L^I(n)} \lambda^{l'}}, & \text{if } k(n) \neq 0 \\ \frac{\lambda^l}{|L_{out}(n)| + |L^I(n)|}, & \text{if } k(n) = 0 \end{cases}, \quad (13)$$

and

$$q_n(\lambda) = \begin{cases} \frac{\sum_{l' \in L_{out}(n)} \lambda^{l'}}{\sum_{l' \in L_{out}(n)} \lambda^{l'} + \sum_{l' \in L^I(n)} \lambda^{l'}}, & \text{if } k(n) \neq 0 \\ \frac{|L_{out}(n)|}{|L_{out}(n)| + |L^I(n)|}, & \text{if } k(n) = 0 \end{cases}, \quad (14)$$

where  $k(n) := \sum_{l' \in L_{out}(n)} \lambda^{l'} + \sum_{l' \in L^I(n)} \lambda^{l'}$ .

Now we are ready to solve the dual problem (8) using a projected sub gradient method. At each node  $n$  for  $L_{out}(n)$  and  $S(l)$ , the outgoing link prices for sources involved are adjusted as follows

$$\lambda_{l_s}(t+1) = \left[ \lambda_{l_s}(t) - \gamma(t) \frac{\partial D}{\partial \lambda_{l_s}}(\lambda(t)) \right]^+, \quad (15)$$

Where  $[a]^+ := \max\{0, a\}$  and  $\gamma(t) > 0$  is a step size. '+' denotes the projection onto the set + of non-negative real numbers.

According to Dan skin's theorem, we have

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$$\frac{\partial D}{\partial \lambda_{l_s}} = \log c_l + \log \alpha_{l_s} + \log p_l + \sum_{k \in N^I(l)} \log(1 - q_k) - z_s. \quad (16)$$

Substituting (16) into (15), we obtain the following adjustment rule for link  $l \in L_{out}(n)$  at each node  $n$

$$\lambda_{l_s}(t+1) = \left[ \lambda_{l_s}(t) + \gamma(t) \left( z_s(\lambda(t)) - \log c_l - \log \alpha_{l_s}(\lambda(t)) - \log p_l(\lambda(t)) - \sum_{k \in N^I(l)} \log(1 - q_k(\lambda(t))) \right) \right]^+. \quad (17)$$

## ALGORITHM

- 1) Create a random network.
- 2) After creating a network, the distance between each node to all other nodes are found by using Euclidean distance method.  
d=sqrt((x1-x2)^2+(y1-y2)^2)
- 3) Find the neighbor list of each node is used.
- 4) The neighboring list in order to find the path between each source and destination using DSR routing protocol is implement.
- 5) Checks for the best optimal path from the various paths obtained in the evaluation of route.

Algorithm at source s

6) Receives from the network the sum  $\lambda^s(t) = \sum_{l \in L(s)} \lambda_{l_s}(t)$  of link prices in s's path;

7) Computes the new log rate using

$$z_s(t+1) = \arg \max_{z_s} (U'_s(z_s) - \lambda^s(t) z_s);$$

8) Communicates the new log rate  $Z_s(t+1)$  to all links  $l \in L(s)$  on s's path.

Algorithm at Node n:

9) Receives log rates  $Z_s(t)$  from all sources  $s \in \cup_{l \in L_{out}(n)} S(l)$  that go through the outgoing links of node  $n$

10) Receives prices  $\lambda^{l'}(t), \forall l' \in L^I(n)$  from the neighboring nodes  $n'$  where  $l' \in L_{out}(n')$ ;

11) Calculates  $\alpha_{l_s}(t), p_l(t), q_n(t), \forall l \in L_{out}(n), \forall s \in L(s)$ , according to Proposition 2;

12) Computes new prices

$$\lambda l_s(t+1) = [\lambda l_s(t) + \gamma(t)(Z_s(\lambda(t)) - \log c_l - \log \alpha_{l_s}(\lambda(t)) - \log p_l(\lambda(t)) - \sum_{k \in N^I(l)} \log(1 - q_k(\lambda(t))))]^+.$$

For each outgoing link  $l \in L_{out}(n)$ , communicates new Prices  $\lambda_{ls}(t+1)$  to all sources  $s \in S(l)$  that use link  $l$  and  $\lambda^l(t+1)$  to all nodes in  $N^l(l)$ . For the convergence and optimality of this distributed algorithm, have the following result.

If the following condition is satisfied at the optimal dual solution  $\lambda^*$

$$k^*(n) = \sum_{l \in L_{out}(n)} \sum_{s \in S(l)} \lambda_{ls}^* + \sum_{l \in L^l(n)} \sum_{s \in S(l)} \lambda_{ls}^* \neq 0, \forall n \in N$$

And  $\lambda^*$  denotes a minimize of the dual problem (8), then step sizes  $\{\gamma(t)\}_{t=0}^{\infty}$  exist to guarantee  $\lim_{t \rightarrow \infty} \lambda(t) = \lambda^*$

### Implementation Issues

**Utility function and global parameter:** The utility function is determined by the objective of the end user such as fairness requirement. The smaller the global parameter  $\gamma$ , the closer does the algorithm converge to the optimal point. Its value can be chosen guided by simulations.

**Congestion price and queuing:** A natural choice of congestion price is queue length. Each node does not need to keep per flow information but distinguishes flows by their destinations.

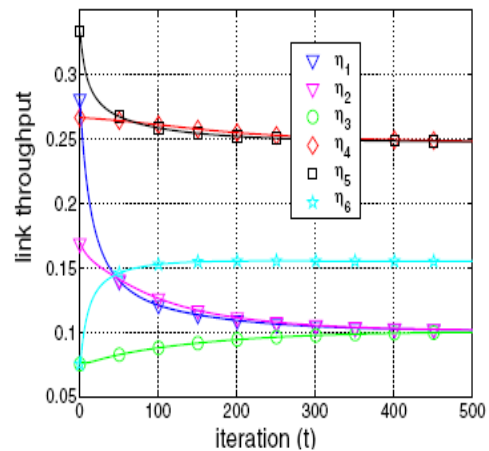
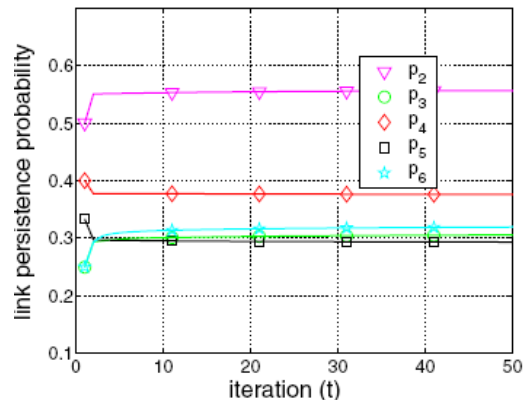
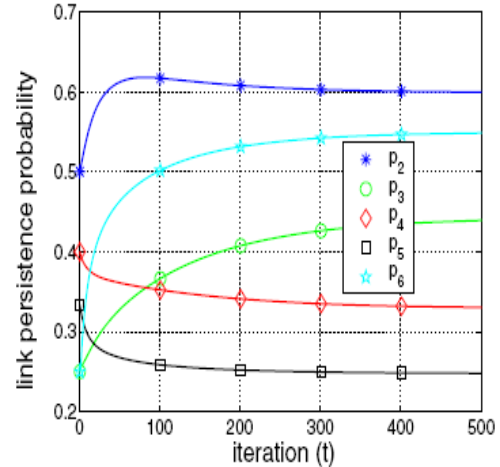
Therefore, each node should manage separate queues for flows going to different destination nodes.

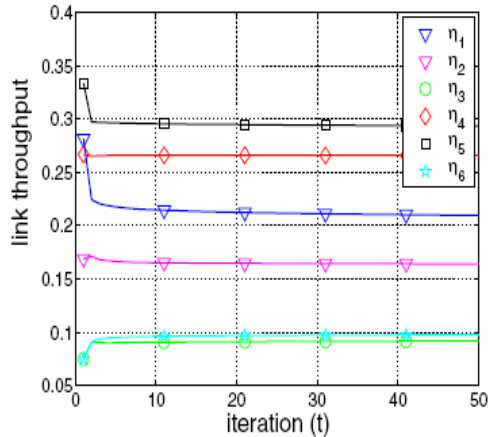
### Message passing and communication overhead:

Each node needs to communicate its congestion price information to its neighbors. This can be achieved by periodically broadcasting this information to its neighbors or its neighbors can actively send inquiring message to ask for this information.

We now examine the implications of our design to the layered and distributed network architecture. Our congestion control is not an end-to-end scheme. Each source node adjusts its sending rate according to the local congestion price. Thus, there is no communication overhead for congestion control. This is very different from the end-to-end congestion control where the “global” aggregate congestion price along the whole path needs to be fed back to the source node. Also, there is no communication overhead for routing, since we basically get routing for free from the scheduling. The majority of communication overhead is for scheduling

**The evolution of link persistence probabilities and link throughputs.**





The joint control algorithm can be implemented as follows. Each link  $l$  (or its transmission node  $tl$ ) updates its persistence probability  $p_l(t)$ , and concurrently, each source updates its data rate  $x_s(t)$ . To calculate the sub gradient in (6), each link needs information only from link  $k$ ,  $k \in L_l$ , i.e., from links whose transmissions are interfered from the transmission of link  $l$ , and those links are in the neighborhood of link  $l$ . To calculate the sub gradient in (7), each source needs information only from link  $l$ ,  $l \in L(s)$ , i.e., from links on its routing path. Hence, to perform the algorithm, each source and link need only local information through limited message passing and the algorithm can be implemented in a distributed way. At the transmitter node of each link to update the persistence probability of that link, and does not need to be passed among the nodes. There is no need to explicitly pass around the values of persistence probabilities

## CONCLUSION

We studied the joint design of congestion and contention control for wireless ad hoc networks. While the original problem is non-convex and coupled, provided a decoupled and dual-decomposable convex formulation, based on which sub gradient-based cross-layer algorithms were derived to solve the dual problem in a distributed fashion for non-logarithmic utilities. These algorithms decompose vertically in two layers, the network layer where sources adjust their end-to-end rates, and the MAC layer where links update persistence probabilities. These two layers interact and are coordinated through link prices. We used random network topology in wireless ad hoc network. In the future, we plan to study the joint congestion control and contention control problem in a hybrid wire line and wireless network and using different topologies evaluate the performance in wireless ad hoc network.

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