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Householder QL-Method and Gerschgorin circle method to compute the eigen values of the tri diagonal matrix

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Abstract— The eigenvalues of the tridiagonal matrix has been computed using the Householder QL-Method .In this paper, for the same system matrix eigenvalues have been computed using Gerschgorin circles by applying secant method at the Gerschgorin bound.

Keywords— *Gerschgorin circles, eigenvalues, system matrix, secant method, Gerschgorin bound.*

I. Introduction

In literature there exist several methods to compute the eigenvalues of the system matrix [2, 3].In Householder QL-Method [1] eigenvalues have been computed for the tridiagonal matrix. In this method first the symmetric matrix has been considered and by this matrix is transformed to tridiagonal matrix using the plane rotations. The transformation is applied for 3 iterations. In this paper a simple Gerschgorin circles [4] have been used to compute the eigenvalues of the matrix. Here secant method is applied at the Gerschgorin bound and the eigenvalues that are obtained are very accurate.

II. Existing Method

Consider a tri diagonal matrix of order (3x3)

$$T_1 = \begin{pmatrix} 4 & 2 & 0 \\ 2 & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

The above matrix is obtained by applying Householder method to some symmetric matrix.

The first step is to evaluate L_1

$L_1 = S_2 S_3 T_1$ Note n = 3, two matrices since k = 2, 3, …… . n

First evaluate $S_2 T_1 = T_1{}'$

For k = 3

1 0 0

$$S_3 = \begin{pmatrix} 0 & \cos\theta_2 & -\sin\theta_3 \\ 0 & \sin\theta_3 & \cos\theta_3 \end{pmatrix}$$

where a_3 and b_3 are taken from T_1

$$\cos\theta_2 = \cos\theta_3 = \frac{a_3}{\sqrt{b_3^2 + a_3^2}} = \frac{2}{\sqrt{(1)^2 + (2)^2}} = 0.894$$

$$\sin\theta_3 = \frac{b_3}{\sqrt{b_3^2 + a_3^2}} = 0.447$$

$$\text{Hence } S_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & .894 & -0.447 \\ 0 & -0.447 & 0.894 \end{pmatrix}$$

$$S_3 T_1 = T_1{}' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & .894 & -0.447 \\ 0 & -0.447 & 0.894 \end{pmatrix} \begin{pmatrix} 4 & 2 & 0 \\ 2 & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 2 & 0 \\ & & \\ & & \end{pmatrix}$$

$$= \begin{matrix} 1.788 & 2.235 & 0 \\ 0.894 & 2.235 & 2.235 \end{matrix}$$

Notice that the elements above the n^{th} diagonal element or (n-1, n) – th element are now reduced to zero. The (n, n-2) element have become non zero,

Next step is to multiply $S_2 T_1^{'} = T_1^{''} = L_1$

For k=2

$$S_2 = \begin{pmatrix} \cos\theta_1 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\cos\theta_1 = \cos\theta_2 = \frac{a_2}{\sqrt{b_2^2 + a_2^2}} \quad \frac{2.236}{\sqrt{(2)^2 + (2.238)^2}} = 0.745$$

$$\text{Hence} \quad S_2 = \begin{pmatrix} 0.745 & -6.67 & 0 \\ 0.667 & 0.745 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$L_1 = S_2 T_1^{'} = T_1^{''}$

$$\begin{pmatrix} 0.745 & -0.667 & 0 \\ .788 & 2.235 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 & 2 & 0 \\ 0.667 & 0.745 & 0 \\ 0.894 & 2.235 & 2.235 \end{pmatrix}$$

$$L_1 = \begin{pmatrix} 1.788 & 0 & 0 \\ 4.0 & 3.0 & 0 \\ 0.893 & 2.235 & 2.235 \end{pmatrix}$$

$Q_1 = S_3^T * S_2^T$

$$Q_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & .894 & -0.447 \\ 0 & -0.447 & 0.894 \end{pmatrix} \begin{pmatrix} 0.745 & -6.67 & 0 \\ 0.667 & 0.745 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.745 & 0.667 & 0 \\ -0.596 & 0.666 & 0.447 \\ 0.298 & -0.333 & 0.894 \end{pmatrix}$$

Now a check is performed to verify $T_1 = Q_1 L_1$

$$T_1 = \begin{pmatrix} 0.745 & 0.667 & 0 \\ -0.596 & 0.666 & 0.447 \\ 0.298 & -0.333 & 0.894 \end{pmatrix} \begin{pmatrix} 1.788 & 0 & 0 \\ 4.0 & 3.0 & 0 \\ 0.894 & 2.235 & 2.235 \end{pmatrix}$$

$$= \begin{pmatrix} 4.0 & 2.0 & 0 \\ 2.0 & 3.0 & 1.0 \\ 0 & 1.0 & 2.0 \end{pmatrix}$$

Now the first transformation can be performed

$T_2 = L_1 Q_1$

$$T_2 = \begin{pmatrix} 1.788 & 0 & 0 \\ 4.0 & 3.0 & 0 \\ 0.894 & 2.235 & 2.235 \end{pmatrix} \begin{pmatrix} 0.745 & 0.667 & 0 \\ -0.596 & 0.666 & 0.447 \\ 0.298 & -0.333 & 0.894 \end{pmatrix}$$

$$\begin{pmatrix} 1.33 & 1.19 & 0 \\ 1.19 & 4.667 & 1.341 \\ 0 & 1.341 & 3.0 \end{pmatrix}$$

Now that the off diagonal elements of T_2 are reduced compared to those in T_1.

T_2 is now factorized into L_2 and Q_2 in the following way

$L_2 = S_2 * S_3 * T_2$

Evaluate $S_3 T_2 = T_2^{'}$ first

For k = 3

$$S_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_2 & -\sin\theta_3 \\ 0 & \sin\theta_3 & \cos\theta_3 \end{pmatrix}$$

where a_3 and b_3 are taken from T_2

$$\cos\theta_2 = \cos\theta_3 = \frac{a_3}{\sqrt{b_2^2 + a_2^2}} = \frac{3.0}{\sqrt{(1.341)^2 + (3)^2}}$$

=0.913

$$\sin\theta_2 = \frac{b_2}{\sqrt{b_2^2 + a_2^2}} = \frac{1.19}{\sqrt{(1.19)^2 + (3.72)^2}} = 0.305$$

$$S_2 = \begin{pmatrix} 0.952 & -0.305 & 0 \\ 0.305 & 0.952 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$L_2 = T_2^{''} = S_2 \; T_2^{'}$

$$\begin{pmatrix} 0.952 & -0.305 & 0 \\ 0.305 & 0.952 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1.33 & 1.19 & 0 \\ 1.090 & 3.71 & 0 \\ 0.490 & 3.13 & 3.29 \end{pmatrix}$$

$$= \begin{pmatrix} 0.93 & 0 & 0 \\ 1.44 & 3.89 & 0 \\ 0.490 & 3.13 & 3.29 \end{pmatrix}$$

The orthogonal matrix Q_2 is formed by multiplying the transpose of the S matrices

$Q_2 = \; S_3^{T} \; * S_2^{T}$

$$Q_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.913 & 0.408 \\ 0 & 0.408 & 0.913 \end{pmatrix} \begin{pmatrix} 0.952 & 0.305 & 0 \\ -0.305 & 0.952 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.952 & 0.305 & 0 \\ -0.278 & 0.869 & 0.408 \\ 0.124 & -0.388 & 0.913 \end{pmatrix}$$

Now a check is performed

$T_2 = L_2 \, Q_2$

$$\begin{pmatrix} 0.952 & 0 & 0 \\ -2.78 & 0.869 & -0.408 \\ 0.124 & -0.388 & 0.913 \end{pmatrix} \begin{pmatrix} 0.93 & 0 & 0 \\ 1.44 & 3.89 & 0 \\ 0.490 & 3.13 & 3.29 \end{pmatrix}$$

Now the next transform is performed

$T_3 = L_2 \, Q_2$

$$\begin{pmatrix} 0.93 & 0 & 0 \\ 1.44 & 3.89 & 0 \\ 0.490 & 3.13 & 3.29 \end{pmatrix} \begin{pmatrix} 0.952 & 0.305 & 0 \\ -2.78 & -0.869 & 0.408 \\ 0.124 & -3.88 & 0.913 \end{pmatrix}$$

The procedure is continued until the off diagonal elements are smaller than a specified amount, and then the diagonal elements give the eigenvalues. After 3 iterations we see that

$\lambda_1 = 0.89$

$\lambda_2 = 3.83$

$\lambda_3 = 4.28$

However the off diagonal elements are still quite large .Therefore more iteration are needed to find a more accurate answer. After 7 iterations we find that

$\lambda_1 = 0.86$

$\lambda_2 = 2.48$

$\lambda_3 = 5.65$

Exact eigenvalues are

$\lambda_1 = 0.85$

$\lambda_2 = 2.48$

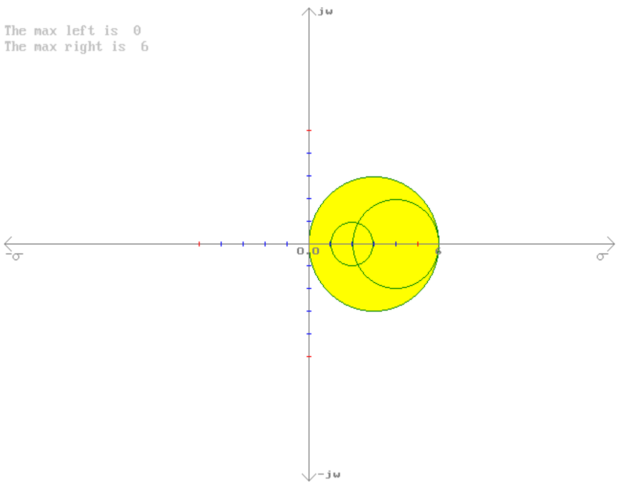
$\lambda_3 = 5.67$

III. PROPOSED METHOD

Consider a tri diagonal matrix of order (3x3)

$$T_1 = \begin{pmatrix} 4 & 2 & 0 \\ 2 & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

Gerschgorin circles of the above matrix are



Gerschgorin bound [0, 6]

Eigenvalues of the above matrix obtained by applying the Secant method at the Gerschgorin bound is

$\lambda_1 = 0.85\,4897$

$\lambda_2 = 2.476024$

$\lambda_3 = 5.669079$

Total computation: 270

IV.CONCLUSION

The existing method as we observe from above requires transformation of matrices, from the symmetric to tridiagonal matrix which is lengthy process .Hence in the method proposed is a simple graphical approach where no transformation is required and the eigenvalues obtained are very exact.

REFERENCES

[1] R.B.Bhat and S. Chakraverty, "Numerical Analysis in Engineering ",Narosa Publishing House Pvt.Ltd. – New Delhi –p.p 210-214.

[2] X. H. Sun et al., "Efficient tridiagonal solvers on multicomputer", *IEEE Transactions on Computers*, Vol. 41, no. 3, 1992, pp. 286-296.

[3] Y. Juang and W. F. McColl," A two-way BSP algorithm for tridiagonal systems", *Future Generation Computer Systems*, Vol.13, Issues 4-5, 1998, pp. 337-347

[4] M. K. Jain, S. R. K. Iyengar and R. K. Jain, "Numerical Methods for Scientific and Engineering Computation", Wiley Eastern Limited, 1993.pp.201-203.

[4] V. S. Pusadkar and S. K. Katti, "A New Computational Technique to identify real eigenvalues of the system matrix A", via Gerschgorin theorem, *Journal of Institution of Engineers (India)*, Vol. 78, Dec. 1997, pp. 121-123.

[5] Y.V. Hote,"New Approach of Kharitonov and Gerschgorin theorem in Control Systems", Ph. D thesis, Delhi University, 2009.

[6] Y. V. Hote D. Roy Choudhury, and J. R. P. Gupta, "Gerschgorin theorem and its applications in control system problems", *IEEE International Conference on Industrial Technology*, 2006, pp. 2438-2443.

[7] T.D.Roopamala and S.K.Katti, "Eigenvalues of tridiagonal matrix using Strum sequences and Gerschgorin Theorem" IJCSE, Vol3.No.12.Dec 2011 .pp.3722-3727.

[8] T.D.Roopamala and S.K.Katti, "Comments on "Computation of range of interval between in which the roots lie by Given's technique ", IJCSNS, Vol.10, No.3.pp.206-.207 March 2010.