Optimal Allocation Techniques for Reducing the Sensing Error Probability with Improved Energy Detection in Cognitive Radio

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Abstract—In cognitive radio, spectrum sensing is one of the important tasks and is used to detect primary users. Energy detection is used for the spectrum sensing when the prior information about the primary users is not available. In this paper an improved version of energy detection is reported by increasing the detection performance. In this work sensing error probability is reduced by using an algorithm to improve the energy detection, here a decision statistic is computed by an arbitrary positive index p in place of squaring operation. An optimal value of threshold is derived which is satisfying a minimum error probability and derived the minimum number of samples required to achieve a target error probability.

Keywords—cognitive radio, spectrum sensing, energy detection, improved energy detection, sensing error probability.

I. INTRODUCTION

Cognitive Radio (CR) [1], [2], [3], [4], [5], [6], [7] has emerged as a promising solution that can effectively deal with the existing conflicts between spectrum demand growth and spectrum underutilization. CR aims at improving spectrum usage efficiency by allowing some unlicensed (secondary) users to access in an opportunistic and non-interfering manner to some licensed bands temporarily unoccupied by the licensed (primary) users.

One of the most important challenges for a CR network is not to cause harmful interference to primary users. To guarantee interference-free spectrum access, secondary users should reliably identify the presence of primary users, which basically aims to determine whether a primary signal is present within a certain frequency range. The spectrum utilization can be improved by allowing an unlicensed or secondary user (SU) to access a licensed frequency band at the time when the licensed or primary user (PU) is absent. In cognitive radio, SU senses an idle frequency band of a PU, and if a band is found to be idle, SU may transmit over that band. But as soon as PU returns, SU must vacate the band immediately. This complete process requires accurate spectrum sensing to avoid harmful interference to PU. There are number of different spectrum sensing techniques that have been proposed so far such as likelihood ratio test [8], matched filtering based detection [9], cyclostationary detection [10], [11] covariance based detection [12], [13] eigenvalue based detection [14], [15] and energy detection (ED) [3], [9], [11], [16], [17], [18]. ED does not require a prior knowledge about primary signals, it is less complex and easy to implement than the other spectrum sensing techniques. In this method the energy of the received signal is compared with a pre-evaluated threshold and it is a non coherent technique. In the most generic case, a CR user is not expected to be provided with any prior information about the primary signals that may be present within a certain frequency band. When the secondary receiver cannot gather sufficient information, the energy detection principle [16] can be used due to its ability to work irrespective of the signal structure to be detected. Therefore energy detection technique is preferred when sufficient information is not available about PUs. While other techniques depend on the detection accuracy, sensing time, information availability about primary signals and a complicated computation work.

In this paper for improved energy detection [19], [20] algorithm, total probability of error is derived. The minimum error probability which is satisfied by the existing optimum threshold is obtained by simulation. An arbitrary suitable positive power is obtained numerically by simulations for obtaining the decision statistic. A target error probability is derived to find out the minimum number of samples required. An optimum value of the arbitrary positive power which minimizes the minimum number of samples is identified. The optimum value of the arbitrary positive power which minimizes the minimum number of samples is derived and verified numerically by simulations.

II. SPECTRUM SENSING PROBLEM FORMULATION

The spectrum sensing problem can be formulated by using two binary hypothesis problem of testing [3], [19] which is presented as follows:

\[ H_0 \text{ stands for: } y(n) = w(n) \rightarrow (PU \text{ unavailable}) \]  
\[ H_1 \text{ stands for: } y(n) = x(n) + w(n) \rightarrow (PU \text{ available}) \]

In this case, \( y(n) \) is the signal received, at the secondary receiver, \( n = 1, 2, 3, \ldots N \) indicates the samples of received signal by secondary user. \( w(n) \) represents the sample of AWGN (Additive White Gaussian Noise) having variance \( \sigma_w^2 \).
and \(x(n)\) is the primary user signal assumed to be real Gaussian with variance \(\sigma^2_x\). Here, \(x(n)\) and \(w(n)\) are independent as per assumption, it is considered that information about noise power is known.

In the binary hypothesis, first expression that is \(H_0\) denotes the absence of primary user and second expression that is \(H_1\) denotes the presence of the primary user. When the spectrum sensing fails then the resulting probability is known as probability of false alarm or probability of missed detection. In case of probability of missed detection, when primary user is available in spectrum band then the technique of spectrum sensing selects hypotheses \(H_0\). In case of probability of false alarm, when the frequency band is not occupying any signal then the technique selects hypotheses \(H_1\). In probability of missed detection, there is a consequence of resulting interference to the primary user. While in probability of false alarm, it results in missing the opportunities and due to which the utilization of spectrum in an effective and efficient manner decreases. Depending on these two definitions of probability, it can be concluded that there are two types of probabilities namely, probability of detection and probability of false alarm. To optimize the performance of cognitive radio it is desired that probability of detection should be as large as possible and probability of false alarm should be minimized.

### III. ENERGY DETECTION

A number of spectrum sensing methods have been proposed in the literature to identify the presence of primary signals during transmission. For practical applications it is important to know information about the primary signal users, but for CR user it is not essential to provide any prior information about the primary signals that may be present within a certain frequency range. When the secondary receiver cannot gather sufficient information about the primary user signal, the energy detection principle can be employed due to its ability to work irrespective of the actual signal to be detected. Due to the simplicity and relevance, energy detection technique has been a preferred approach for many spectrum sensing studies and this approach is adopted in this research work. An energy detector, simply measures the energy received on a primary band during an observation period and declares the band as occupied if the measured energy is greater than a properly set threshold, or unoccupied otherwise [11], [21], [22], [23], [24]. Thus, the test statistic \(T_{CED}\) for the conventional energy detector is given by:

\[
T_{CED} = \frac{1}{N} \sum_{n=1}^{N} |y(n)|^2
\]

where \(T_{CED}\) is the test statistic and \(N\) is the number of samples used for computation. The test statistic \(T_{CED}\) is compared with a pre-evaluated threshold \(\gamma\). If \(T_{CED} > \gamma\), the decision is hypothesis \(H_0\), otherwise hypothesis \(H_1\). Song et al. [13] has given the probability density function of \(T_{CED}\) which is given as follows:

\[
f_{T|H_0}(y) = \frac{1}{2^N(\Gamma(N/2))^{N/2}} \exp \left( -\frac{\lambda}{2} \right)
\]

\[
f_{T|H_1}(y) = \frac{1}{\alpha} \exp \left( -\frac{\lambda + \alpha}{2} \right) I_{N/2}(\sqrt{\alpha T})
\]

Where \(\alpha = \frac{\sigma^2_w}{\sigma^2_x}\) is the SNR, \(\Gamma()\) is the complete Gamma function and \(I_m()\) is the \(m^{th}\) order Bessel function of the first kind. The central limit theorem is applied and it is observed that as \(N\) increases for the test statistic the normal distribution is followed approximately. The probability density function of \(T_{CED}\) is then given by,

\[
f_{T|H_0}(y) = \frac{1}{\sqrt{2N\sqrt{\pi}}} T_{CED}^{N-1} \exp \left( -\frac{(\lambda - N)^2}{2(2N)} \right)
\]

\[
f_{T|H_1}(y) = \frac{1}{\sqrt{2N(1 + \alpha)\sqrt{\pi}}} \exp \left( -\frac{(\lambda - N(1 + \alpha))^2}{2(2N(1 + \alpha))} \right)
\]

Receiver operating characteristics (ROC) curve are used to depict the relationship between these values and are expressed as follows,

\[
P_d = Q \left[ \frac{\sqrt{2N}Q^{-1}(P_f) - N\alpha}{\sqrt{2N(1 + \alpha)}} \right]
\]

\[
P_f = Q \left[ \frac{\sqrt{2N(1 + \alpha)(P_d)Q^{-1} - N\alpha}}{\sqrt{2N}} \right]
\]

The total error probability depends on the values of \(P_d, P_f\) and the probability of occurrence of \(H_0\) and \(H_1\). The total error probability is denoted by \(P_{ep}\) and given as follows:

\[
P_{ep} = (1 - P)f + P(1 - P_d)
\]

where \(P\) is the probability of occurrence of the primary user.

### IV. IMPROVED ENERGY DETECTION

Improved energy detector [19], [20] for random signals in Gaussian noise is proposed by replacing the squaring operation of the signal amplitude in the conventional energy detector with an arbitrary positive power operation. The decision statistic of the improved energy detector \(T_{IED}\) with \(p^\theta\) power summer is given by
where $T_{IED}$ is the decision statistic and $N$ is the number of samples used for computation. The decision statistic $T$ is compared with $\gamma_m$, therefore the modified decision threshold is to distinguish between the two hypotheses:

$$T_{IED} \begin{cases} > \gamma_m & H_1 \\ < \gamma_m & H_0 \end{cases}$$

For $p = 2$, the improved energy detection becomes the conventional energy detection. For any value of $p$, $|y(n)/\sigma_n|^p$ are independent and identically distributed random variables. Considering the Ref [19], under hypothesis $H_0/H_1$ the mean and variance of $|y(n)/\sigma_n|^p$ is given by,

$$\mu_{0|H0} = \frac{2^{p/2}}{\sqrt{\pi}} \Gamma\left(\frac{p+1}{2}\right)$$  \hspace{1cm} (13)$$

$$\mu_{1|H1} = \frac{2^{p/2}(1+\alpha)^p}{\sqrt{\pi}} \Gamma\left(\frac{p+1}{2}\right)$$  \hspace{1cm} (14)$$

$$\sigma_{0|H0}^2 = \frac{2^p}{\sqrt{\pi}} \left\{ \Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}} \Gamma^2\left(\frac{p+1}{2}\right) \right\}$$  \hspace{1cm} (15)$$

$$\sigma_{1|H1}^2 = \frac{2^p(1+\alpha)^p}{\sqrt{\pi}} \left\{ \Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}} \Gamma^2\left(\frac{p+1}{2}\right) \right\}$$  \hspace{1cm} (16)$$

V. SIMULATIONS AND ANALYSIS

In figures (1), (2) and (3) the Receiver operating characteristics (ROC) curve of the conventional energy detector is compared with the improved energy detector and improvement is observed in the enhanced algorithm over the conventional algorithm.

![Fig.1: Receiver operating characteristics (ROC) curves of the proposed and existing algorithms (SNR= -15dB, N=1000)](image1)

![Fig.2: Receiver operating characteristics (ROC) curves of the proposed and existing algorithms (SNR= -10dB, N=500)](image2)
In figure (4) the performance of the proposed algorithm with optimum threshold for varying SNR is compared with the existing algorithm. It is observed that, at the negative value of SNR, the total error probability is at a higher side and depending also on the number of samples. But it’s value is lower than the values obtained with the existing improved energy detection algorithm. Further it is observed that at the values of SNR greater than zero, the value of total error probability decreases up to zero for the proposed algorithm when it is compared with the existing algorithm.

Figure.4. Performance of the proposed and existing algorithms against SNR

In figure (5) samples complexity is compared between proposed and existing algorithm.

Figure.5. Sample Complexity of the proposed and existing algorithms.
The minimum number of samples required for the existing algorithm is given by Urkowitz. H. [16] and M. Lopez-Benitez et al. [20].

$$N_{\text{min}} = 2^\frac{1}{\alpha^2} \left( Q^{-1} \left( P_\alpha \right) - Q^{-1}(P_\alpha)\sqrt{2\alpha + 1} \right)$$ (17)

From equation (20), it is found that the sample complexity is in the order of $1/\alpha^2$. In the figure, (5) the curve is plotted between the number of samples required at different value of SNR for the proposed and existing algorithms numerically. For both the cases the target probability of false alarm is set to (0.1). For the proposed algorithm target probability is set to be (0.15). The curves for the proposed and existing algorithm the curve shifts downwards slightly. It shows that the sample complexity of the proposed algorithm also scales to the order of $1/\alpha^2$. Further it is noticed that for the low values of SNR i.e ($<-5$dB), less number of samples are required for proposed algorithm in comparison to traditional energy detection algorithm. But both the algorithms requires very less number of samples for SNR value greater than (-5dB). Therefore it is easy to detect the signal in this region with less number of samples for any signal detection algorithm.

VI. CONCLUSIONS

Energy detector method has become popular due to its simplicity and less computational work. In this paper application of improved energy detector is proposed to improve the performance of spectrum sensing. An optimal value of threshold is derived which is satisfying a minimum error probability and derived the number of samples required to achieve target error probability. The total sensing error probability is the parameter considered for the design of the proposed techniques. Simulation results validate the efficiency of the proposed algorithms therefore it is better than the existing algorithm and may be used for practical application of spectrum sensing.

REFERENCES


